

Teaching and application of exercises on Planck quantities

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Abstract: *The work presents the treatment of Planck's quantities in the classroom, where the research comprised part of one of the works carried out on the "Study of the quantization of space-time applied to the entropy of black holes" with 23 students of the 3rd year of high school in São Cristóvão, Rio de Janeiro, Brazil. The project is part of several subjects introduced on modern and contemporary physics, as well as frontier research themes, aiming at the student-researcher posture and based on the learning theories of Ausubel and Bruner, develop significant learning and receive new information and concepts at an elementary level, conditioning for a critical and reflective posture. We carried out the activity in an expository way and with the application of questions on the topic addressed, surveying data to verify the performance of the students.*

Keywords—Planck; quantization; temperature; mass; energy.

1. INTRODUCTION

The introduction of modern physics topics, among other frontier topics, will always be challenges in the teaching process, as the teacher must prepare and understand the approach very well, as well as develop a mechanism for transmitting this knowledge and its application to achieve the desired effect, which learning by students.

We developed a study on Planck quantities and carried out this research with 23 students of the 3rd year of high school in São Cristóvão, Rio de Janeiro, Brazil, with the objective of presenting how physics deals with these quantities to start the quantization processes.

The work is part of a dissertation research with the theme "Study of quantization applied to the entropy of black holes" [1], with a theoretical foundation of learning based on Ausubel, considering meaningful learning and Bruner who considers the ability to deal with any topic, but with the means and languages necessary for the audience for which it is intended [2].

It sought to develop scientific skills, meaningful learning, dialogicity and criticality, with the involvement of research projects in the classroom, working on topics of modern physics and frontier research subjects [3,4]. Whenever possible, we seek to change the routine in the classroom, with innovative themes, always valuing the student's thinking and interaction with the teacher [5].

The article is presented with the theoretical foundation, where we will address Planck's magnitudes, following with the methodology, where it shows how it was applied and the methods used, as well as the analysis and discussion of the results, with the data of the exercises carried out by the students.

2. THEORETICAL DISCUSSION

2.1 Planck quantities

Planck's quantities are important for the study or research for systems that need a quantization look, which is the case of quantum physics, where very small measurements are treated that we cannot perceive as the microscopic world. So, since Planck scales are associated with the smallest physically realizable quantities in the universe, we need to know these units.

The Planck length is the shortest existing length in nature and is where quantum effects are noticeable in the microscopic world, i.e. in the world of quantum mechanics. As we are interested in Planck's constant and its quantities, such as length, mass, energy, time and temperature, Planck realized that the units of the International System (SI), meter (m), kilogram (kg) and second (s), are combinations of three fundamental constants of nature: the speed of light (c), the constant of Universal Gravitation (Newtonian constant) (G) and h Planck's constant introduced by himself. He realizes that we have quantum (h), relativistic (c) and gravitational (G) phenomena. As we will also have the treatment of thermodynamic effects, we will have Boltzmann's constant (k_B) which is the constant that relates temperature and energy of molecules [6].

For numerical values of these constants, we have $G = 6.67 \cdot 10^{-11} \text{ Nm}^2\text{kg}^{-2}$, $c = 3.00 \cdot 10^8 \text{ m/s}$, $k_B = 1.38 \cdot 10^{-23} \text{ JK}^{-1}$ e $h = 6.63 \cdot 10^{-34} \text{ Js}$, in the case of Planck's constant, we will use the reduced form $\hbar = 1.05 \cdot 10^{-34} \text{ Js}$. Planck's constant in electron-volt-second is of the form $h = 4.13 \cdot 10^{-15} \text{ eVs}$.

To arrive at Planck's magnitudes, in the end only Newton's, light's and Planck's constants will remain. Only for Planck's temperature, in addition to the constants mentioned above, we will have Boltzmann's constant, and to calculate Planck's length, we must relate the Law of Universal Gravitation to Newton's 2nd Law, so $F_g = F$, like this

$$\frac{GMm}{d^2} = m \cdot a \rightarrow \frac{GM}{d^2} = \frac{v}{t} \rightarrow v^2 = \frac{GM}{d} \quad (1)$$

And equating Einstein's Energy $E = m \cdot c^2$ with the energy of Eq. (1), We have

$$Mc^2 = hf \rightarrow Mc^2\lambda = hc, \\ \text{with } d = \lambda = l_p \text{ e } v = c \quad (2)$$

Doing the substitution relationship, we find

$$l_p = \sqrt{\frac{\hbar G}{c^3}} \quad (3)$$

For the calculation of Planck time t_p is equal to the time it would take a photon traveling at the speed of light at a distance equal to the Planck length. So doing $t = d/v$, what gives $t = l_p/c$, Have

$$t_p = \sqrt{\frac{\hbar G}{c^5}} \quad (4)$$

Planck's mass M_p is the mass that relates Einstein's energy to that of the photoelectric effect of Eq. (2). By isolating the dough and making the substitutions, we get

$$M_p = \sqrt{\frac{\hbar c}{G}} \quad (5)$$

For Planck energy E_p , which is the rest energy corresponding to the Planck mass, we have $E_p = M_p c^2$, making the replacement of the dough with the Eq. (5), We have

$$E_p = \sqrt{\frac{\hbar c^5}{G}} \quad (6)$$

For Planck temperature, we relate the kinetic energy of the molecules $E_c = \frac{3}{2} k_B T$, with Einstein Energy $E = m \cdot c^2$. Making the substitution with Planck dough, we have for the temperature the shape

$$T_p = \sqrt{\frac{\hbar c^5}{G K_B^2}} \quad (7)$$

Where for the length we got $1.6 \cdot 10^{-35} \text{ m}$, time $5.4 \cdot 10^{-44} \text{ s}$, mass $2.2 \cdot 10^{-8} \text{ kg}$, energy $1.956 \cdot 10^9 \text{ J}$ and temperature $1.4 \cdot 10^{32} \text{ K}$.

3. METHODOLOGY

The application of the questions was carried out with 23 students from the 3rd year of high school, extracting

information not only from the exercises, but also from the students' behavioral observation.

We deal with the theoretical discussion and at the end of the day, we apply the exercises, according to the questions below.

1. Knowing that Planck's temperature is $1,4 \cdot 10^{32} \text{ K}$, length $1,6 \cdot 10^{-35} \text{ m}$, time $5,4 \cdot 10^{-44} \text{ s}$, mass $2,2 \cdot 10^{-8} \text{ kg}$ and energy $1,9 \cdot 10^9 \text{ J}$. Answer the following questions:

a) if the temperature inside the Sun is $1,36 \cdot 10^7 \text{ K}$, how many times is Planck temperature higher?

b) a Camponotus ant even measures $2,5 \cdot 10^{-2} \text{ m}$. How many Planck lengths will it take to reach the measurement of this ant?

c) the Light travels to $3 \cdot 10^8 \text{ m/s}$. How long from Planck would light travel at a distance of $6 \cdot 10^{30} \text{ m}$?

d) Consider the mass of a drop of water with $0,03 \text{ grams}$. How many Planck masses would we get in the mass of a drop of water?

e) Consider the energy of a cosmic ray with $2 \cdot 10^3 \text{ J}$. How much cosmic ray in this energy would it take to reach Planck's energy?

Answer to questions

1) a) $1,03 \cdot 10^{25}$. b) $1,56 \cdot 10^{33}$. c) $3,7 \cdot 10^{65}$. d) $1,36 \cdot 10^3$. e) $9,5 \cdot 10^5$.

4. RESULTS AND DISCUSSION

The question applied involves accuracy on the part of the students in the development of the approximations of the calculations. The deficiency in the treatment of exponential numbers, powers of ten and orders of magnitude is perceived, that is, mathematical tools that would be natural for high school students and that even so, these errors appear in the development of activities.

Item (a) of the question deals with the comparison of the temperature inside the Sun with the Planck temperature. The Planck temperature is obviously higher, and the objective would be how much higher this temperature is in relation to the interior of the Sun. About 78% of the students got the answer closest to what was expected.

Item (b) asks how many Planck lengths would be needed to reach the size of the ant Camponotus and we obtained a percentage of 74% of correct answers.

In item (c) it asks how long Planck light would travel a given distance, and we got 70% of correct answers.

For items (d) and (e), we had a percentage of 61% in relation to the amount of mass of the water drop and 83% in

relation to the energy of cosmic rays, according to the statement of the question.

It can be seen that the problem lies in the mathematical operation, where instead of dividing; some students multiplied and became complicated with the exponential treatment.

5. FINAL CONSIDERATIONS

The students realized that Max Planck spent many years studying Blackbody Radiation and that Planck believed that light propagates according to Maxwell's classical theory of electromagnetism, and that the quantization of radiation occurred only when radiation interacts with matter. In view of Planck's distress, he introduced the constant that bears his name and years later, abandoning the wave model of radiation, he believed that quantization only existed in the cavity region.

The students developed in this work, the ability to question and think critically about reality and work as a team, discussing the issues presented by the teacher, presenting a high percentage of assertiveness, showing interest in participating in the activities.

The objective is to gradually introduce themes of modern and contemporary physics, especially aspects related to quantization, as we approach classical physics in the classroom and limit ourselves only to the treatment of this regime.

The subject of Planck's quantities becomes extremely important for further work, in view of its usefulness and importance to the microscopic world.

6. REFERENCES

- [1] Silva, Jizreel. P. (2022). Study of space-time quantization applied to the entropy of black holes. Rio de Janeiro: Dissertation, Department of Physics, Federal University of the State of Rio de Janeiro.
- [2] Moreira, M. A. (1999). Learning theories. São Paulo: EPU.
- [3] Moreira, M. A. (2021). Challenges in teaching physics. *Revista Brasileira de Ensino de Física*, vol. 43, e20200451, pp.68-73. DOI: <https://doi.org/10.1590/1806-9126-RBEF-2020-0451>.
- [4] Stecanela, N. (2015). Classroom methodology in teacher training and performance. *Revista Pedagógica*, 17(35), 163-178. <https://doi.org/10.22196/rp.v17i35.3060>
- [5] Zacca, A., Santos, A. C. T., & Goulart, L. B. (2012). Working with research in the classroom. *Revista e-Ped*, 2(1), 130-143.
- [6] Santos, C. M. F., Silva, J. M. C., Gomes, E. C., & Lobo, M. P. (2020). A didactic-mathematical proposal for the use of the Planck scale: from photons to black holes. *Revista Brasileira de Ensino de Física*, 42(1), e20190350. <https://doi.org/10.1590/1806-9126-RBEF-2019-0350>