**International Journal of Academic Multidisciplinary Research (IJAMR)**
**ISSN: 2643-9670**
**Vol. 10 Issue 3 March - 2026, Pages: 162-170**

# The group's table of Rational valued characters ($Q_{2m}\times C_9$) When $m=2^h$, $h\in Z^+$

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***Abstract:*** *This topic examines the table of characters with rational values for the group ($Q_{2m}\times C_9$). $Q_{2m}$ represents the generalized Quaternion group of order 2m, while $C_9$ represents the cyclic group of order 9. The symbol (×) denotes the direct product between the two groups. In this case, $m = 2^h$, where h is a positive integer and m is an even number. This topic focuses on finding the irreducible table of characters for the set resulting from the direct product and studying characters whose values are exclusively rational. This is achieved by relying on the properties of the direct product of groups. The characters of the set ($Q_{2m}\times C_9$) can be obtained by multiplying the characters of both groups ($Q_{2m}$ and $C_9$). Analyzing these characters helps in understanding the structure of the group and its representations, which is an important topic in the theory of finite set representations.*

**Keywords— *Quaternion; Cyclic; table; Group; Rational; Characters.***

## Introduction

Group theory is a fundamental and crucial field of mathematics, notably in representation theory, which covers studying groups using matrices and character groups. Character tables are recognized vital devices for knowledge group framework and the features of their representations, as they offer invaluable intelligence with respect to non- reducible representations and its values. In this setting, I. M. Isaacs [3] explored Character Theory of Finite Groups in 1976, while A.H. Abdul-Mun'em [1] in 2008 studies The Artin cokernel of the Quaternion Group Q2m where m is an odd number.

This research deals with the character table with relative values for the group *($Q_{2m}\times C_9$)*, where ($Q_{2m}$) indicates a extended Quaternion group and ($C_9$) denotes the cyclic group of order 9. The study centers on the notable case where ($m = 2^h$), where (h) is a positive integer, which affects the group structure and the properties of characters group.

The present study is designed to consider the non- reducible characters of this group and detect the table of character. This examination helps to augmenting the understanding of the structure of this group and its algebraic properties, through concentrating on characters whose values are rational numbers.

The paper highlights both theoretical and applied dimensions to give a broad and inclusive approach on the present subject.

focusing on characters whose values are rational numbers, which contributes to a deeper understanding of the structure of this group and its algebraic properties. This paper addresses both theoretical and practical aspects.

## Theoretical Aspect

### 1. General Introduction to Group Theory [7]

Group theory is a fundamental to current mathematics, providing an abstract basis for the study of symmetries and algebraic structures. It originally started from the research of polynomial equations which has since grown to cover an extensive range of fields among them theoretical physics, chemistry, number theory, and coding theory.

A group G is a non-empty set equipped with a binary operation that satisfies the following conditions:

- Closure: If $a,b\in G$ , then $ab\in G$.
- Associativeness: $(ab)c = a(bc)$.
- Presence of the identity element: There exists an element $e\in G$ such that $ae = ea = a$.
- Presence of the inverse: For every $a\in G$ , there exists $a^{-1}\in g$.

If the number of elements in a group is finite, it is called a finite group, and the number of its elements is called the group rank.

### 2.Finite Groups and Conjugation Classes [2]

A fundamental concept in the study of finite groups is that of conjugation classes. Two elements are said to be $a, b \in G$ conjugated if there exists an element $g \in G$such that:

$$b = gag^{-1}$$

This relationship divides the elements of a group into distinct groups called conjugation classes, which play a pivotal role in constructing character tables.

The number of conjugation classes equals the number of irreducible representations, a fundamental consequence of representation theory.

### 3. Representation Theory [5]

Representation theory aims to study groups by representing their elements as matrices, allowing the use of linear algebra tools to factor them.

A representation is defined as the application of:

$$T: G \to GL(n, F)$$

such that:

$$T(ab) = T(a)T(b)$$

where $GL(n, F)$ is the invertible matrix group.

#### 3.1 Irreducible Representations

A representation is called irreducible if it is not possible to find a non-zero, constant subspace under the influence of all elements of the group.

These representations are the building blocks from which all other representations are constructed.

### 4.Characters [6]

A character is a function defined on a group in the form:

$$\chi(g) = Tr(T(g))$$

where $T(g)$ is an array representing the element $g$, and Tr means trace.

#### 4.1 Character Properties

- Consistent on conjugate classes
- Achieves orthogonality relations
- Used to distinguish representations

### 5. Characters with Rational Values [8]

A character $\chi$ is said to have rational values if:

$$\chi(g) \in \mathbb{Q} \ \forall g \in G$$

#### 5.1 Importance of Rational Characters

- They are related to Galois theorem
- They simplify representation analysis
- They help in group classification

#### 5.2 The Galois Group and its Influence

If we have a generative field of character values, the Galois group influences these values, and by combining the conjugates, we obtain relative characters.

### 6. Character Table [4]

A character table is an essential tool that displays all irreducible characters against their conjugate classes.

#### 6.1 Table Structure

- Rows: Characters
- Columns: Conjugate Classes

#### 6.2 Importance of the Table

- Determining the number of representations
- Analyzing the algebraic structure
- Studying symmetries

### 7. The Generalized Quaternion $Q_{2m}$ [3]

The generalized quaternion is an important example of a non-Abelian group. It is defined by the relationship:

$$Q_{2m} = < x, y \mid x^m = 1, y^2 = x^{m/2}, yxy^{-1} = x^{-1} >$$

**International Journal of Academic Multidisciplinary Research (IJAMR)**
**ISSN: 2643-9670**
**Vol. 10 Issue 3 March - 2026, Pages: 162-170**

### 7.1 Its properties
- Non-Abelian
- Contains elements of different orders
- Has a relatively complex structure

### 7.2 Its importance
- Used in the study of representations
- A model for non-commutative groups

## 8. Cyclic group $C_9$ [3]
This is an Abelian group generated by a single element:

$$C_9 = \langle z | z^9 = 1 \rangle$$

### 8.1 Properties
- All its elements are powers of a single generator
- Easily factored
- All its representations are first-degree

## 9. Direct product of groups [8]

If $G_1$ and $G_2$ are two groups, then:

$$G_1 \times G_2 = \{(g_1, g_2)\}$$

### 9.1 Properties of the product
- Order = Product of the orders
- If the two groups are Abelian → the result is Abelian

## 10.Direct Product Representations [8]
If:
- $T_1$ is a representation of $G_1$
- $T_2$ is a representation of $G_2$

then:

$$T = T_1 \otimes T_2$$

A is a representation of the direct product.

## 11. Direct Product Characters [4]

$$\chi(g_1, g_2) = \chi_1(g_1)\chi_2(g_2)$$

## 12. Rational Characters of the Direct Product [4]
Through Galois's theorem:

$$\theta = \sum_{\sigma \in Gal} \sigma(\chi)$$

We obtain relative characters from the regular characters.

13. Application to $Q_{2m} \times C_9$

In this research, we:
- Find the characters of the resulting group
- Prove that:
- $\equiv^* (Q_{2m} \times C_9) = \equiv^* (Q_{2m}) \otimes \equiv^* (C_9)$

This demonstrates that:
The structure of the table depends directly on the tables of the two original groups.

## 14. Significance of the Results
The significance of this work lies in:
1. Simplifying the calculation of character tables
2. Providing a general formula that can be used for other groups
3. Linking:
- Group Theory
- Representation Theory

- Galois Theorem

**15. Orthogonality Relations [8]**

Orthogonality relations are among the most important findings in character theory, as they help verify the correctness and completeness of the character table.

**16.1 Row Orthogonality**

If $\chi_J$ and $\chi_i$ are non-reducible characters, then:

$$\frac{1}{|G|}\sum_{g\in G}\chi_i(g)\overline{\chi_\iota(g)} = \begin{cases}1 & i = j \\ 0 & i \neq j\end{cases}$$

**16.2 Perpendicularity between columns**

$$\frac{1}{|G|}\sum_{i}\chi_i(g)\overline{\chi_\iota(h)} = \begin{cases}0 & g \nsim h \\ \frac{|G_{(g)}|}{|G|} & g \sim h\end{cases}$$

The importance of these relationships

- valifable results detection
- errors in tables
- used in theoretical conclusions

**17.Γ Gamma-Classes [5]**

Gamma-classes $\Gamma$ are used instead of conjugation classes in the case of rational characters.

Definition

They are a combination of some associated conjugation classes under the influence of the Galois group.

Importance

- Reduces the number of columns in the table
- Simplifies the analysis

**18. Number of Rational Characters**

Important results:

Number of rational characters = Number of gamma-classes

This gives:

- An important verification criterion
- A direct method for calculating the number of rows in the table

**19. Analysis of the gamma-group structure** $Q_{2m} \times C_9$

**19.1 Group rank**

$$|Q_{2m} \times C_9| = |Q_{2m}|.|C_9| = 2m \times 9$$

**19.2 Number of conjugation classes**

Equals the product of:

- Number of gamma-classes $Q_{2m}$
- Number of gamma-classes $C_9$

**19.3 Number of representations**

Equals the number of conjugation classes.

**20. Character behavior under multiplication**

Important results in paper:

$$\theta_{(i,j)} = \theta_i.\theta_j$$

Interpretation

This means that:

The spectral structure of the composite group

Is the product of the two original structures. This is very important theoretically.

## 21. Algebraic Interpretation of Results

Result:

$$\equiv^* (Q_{2m} \times C_9) = \equiv^* (Q_{2m}) \otimes \equiv^* (C_9)$$

This means that:

- Direct product preserves the representational structure
- There are no "distortions" in the characters

**Practical Aspect**

1. The rational valued characters table of the group ($Q_{2m}\times C_9$) when $m=2^h$, $h \in Z^+$ is equal to the tensor product of the rational valued characters table of $C_9$ and the rational valued characters table of $Q_{2m}$ when $m=2^h$, $h \in Z^+$. This means that $\overset{*}{\equiv}(Q_{2m}\times C_9)=\overset{*}{\equiv}(Q_{2m})\otimes\overset{*}{\equiv}(C_9)$.

Proof:

$C_9 = \{1, z, z^2, z^3, z^4, z^5, z^6, z^7, z^8\}$,

Since

$\equiv C_9 = \overset{*}{\equiv} (C_9) =$

**Table (1)**

| | $h_1'$ | $h_2'$ |
|---|---|---|
| $\chi_1'$ | 8 | -1 |
| $\chi_2'$ | 1 | 1 |

Where $h_1' = \{(I)\}$, $h_2' = \{z, z^2, z^3, z^4, z^5, z^6, z^7, z^8\}$, then,

$\chi_1'(h_1') = \theta_1'(h_1') = 8$

$\chi_1'(h_2') = \theta_1'(h_2') = -1$

$\chi_2'(h_1') = \chi_2'(h_2') = \theta_2'(h_1') = \theta_2'(h_2') = 1$

From the definition of $Q_{2m}\times C_9$ , and theorem (1.2) we have

$(\equiv Q_{2m}\times C_9) = (\equiv Q_{2m})\otimes(\equiv C_9)$

Each element in $Q_{2m}\times C_9$

$h_{ng} = h_n . h_g' \ \forall h_n \in Q_{2m}$, $h_g' \in C_8$, $n = 1,2,3, \ldots ,9m$ , $g \in \{I, z, z^2, z^3, z^4, z^5, z^6, z^7, z^8\}$

each irreducible character of $Q_{2m}\times C_9$ is $\chi_{(i,j)} = \chi_i . \chi_j'$

where $\chi_i$ is an irreducible character of $Q_{2m}$ and $\chi_j'$ is the irreducible character of $C_9$, then

$$\chi_{(i,j)}(h_{ng}) = \begin{cases} 8\chi_i(h_n) \ \ if \ j = 1 \ and \ \ g \in \{I\} \\ \chi_i(h_n) \ if \ \ \ j = 2 \ \ and \ \ \ g \in \{I, z^2, z^4, z^6, z^8\} \\ -\chi_i(h_n) \ \ if \ \ j = 2 \ \ and \ \ \ g \in \{z, z^3, z^5, z^7\} \end{cases}$$

From proposition (1.4)

$$\theta_{(i,j)} = \sum_{\sigma \in Gal(Q(\chi_{(i,j)})/Q)} \sigma(\chi_{(i,j)})$$

Where $\theta_{(i,j)}$ is the rational valued character of ($Q_{2m}\times C_9$ ) Then,

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_{(i,j)}(h_{ng}))/Q)} \sigma(\chi_{(i,j)}(h_{ng}))$$

(a) If $j = 1$ and $g \in \{I\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(2\, \chi_i(h_n))$$

$$=2 \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n)) = \theta_i\,(h_n)\ \ .8 = \theta_i\,(h_n)\ .\ \theta_j'\,(h_g')$$

**International Journal of Academic Multidisciplinary Research (IJAMR)**
**ISSN: 2643-9670**
**Vol. 10 Issue 3 March - 2026, Pages: 162-170**

(b) If $j = 1$ and $g \in \{z^2, z^4, z^6, z^8\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(-2\,\chi_i(h_n)) =$$

$$-2 \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n)) = \theta_i\,(h_n)\,.-1 = \theta_i\,(h_n)\ .\theta_j^{'}\,(h_g^{'})$$

(c) If $j = 1$ and $g \in \{z, z^3, z^5\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\,0\chi_i(h_n)) = 0\,. \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n))$$

$$= \theta_i\,(h_n).0 = \theta_i\,(h_n)\ .\theta_j^{'}\,(h_g^{'})$$

where $\boldsymbol{\theta_i}$ is the rational valued character of $Q_{2m}$.

(I) If $j = 2$ and $g \in \{I, z^3, z^5\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n)) = \theta_i\,(h_n)\,.1$$

$$= \theta_i\,(h_n)\ .\theta_j^{'}\,(h_g^{'})$$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(-\chi_i(h_n))$$

$$= -\sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n))$$

$$= \sum_{\sigma \in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n))\,.-1 = \theta_i\,(h_n)\,.-1 = \theta_i\,(h_n)\,.\theta_j^{'}\,(h_g^{'})\,.$$

From [I], [II] we have

$\theta_{(i,j)} = \theta_i.\theta_j'$ .

Then $\equiv^*(Q_{2m} \times C_9) = \equiv^*(Q_{2m}) \otimes \equiv^*(C_9)$ .

**2. Example**

To find $\equiv^*(Q_{16} \times C_9)$ by using the Proposition (2.1) we obtain the following table:

$\equiv^*(Q_{16} \times C_9) =$

**Table (2)**

| Γ classes | $[1, I]$ | $[x^8, I]$ | $[x^4, I]$ | $[x^2, I]$ | $[x, I]$ | $[y, I]$ | $[xy, I]$ | $[1, z]$ | $[x^8, z]$ | $[x^4, z]$ | $[x^2, z]$ | $[x, z]$ | $[y, z]$ | $[xy, z]$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\theta_{(1,1)}$ | 64 | -64 | 0 | 0 | 0 | 0 | 0 | -8 | 8 | 0 | 0 | 0 | 0 | 0 |
| $\theta_{(2,1)}$ | 32 | 32 | -32 | 0 | 0 | 0 | 0 | -4 | -4 | 4 | 0 | 0 | 0 | 0 |
| $\theta_{(3,1)}$ | 16 | 16 | 16 | -16 | 0 | 0 | 0 | -2 | -2 | -2 | 2 | 0 | 0 | 0 |
| $\theta_{(4,1)}$ | 8 | 8 | 8 | 8 | -8 | -8 | 8 | -1 | -1 | -1 | -1 | 1 | 1 | -1 |
| $\theta_{(5,1)}$ | 8 | 8 | 8 | 8 | 8 | -6 | -8 | -1 | -1 | -1 | -1 | -1 | 1 | 1 |
| $\theta_{(6,1)}$ | 8 | 8 | 8 | 8 | -8 | 8 | -8 | -1 | -1 | -1 | -1 | 1 | -1 | 1 |
| $\theta_{(7,1)}$ | 8 | 8 | 8 | 8 | 8 | 8 | 8 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| $\theta_{(1,2)}$ | 8 | -8 | 0 | 0 | 0 | 0 | 0 | 8 | -8 | 0 | 0 | 0 | 0 | 0 |
| $\theta_{(2,2)}$ | 4 | 4 | -4 | 0 | 0 | 0 | 0 | 4 | 4 | -4 | 0 | 0 | 0 | 0 |
| $\theta_{(3,2)}$ | 2 | 2 | 2 | -2 | 0 | 0 | 0 | 2 | 2 | 2 | -2 | 0 | 0 | 0 |
| $\theta_{(4,2)}$ | 1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 |
| $\theta_{(5,2)}$ | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 |
| $\theta_{(6,2)}$ | 1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 |
| $\theta_{(7,2)}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

**International Journal of Academic Multidisciplinary Research (IJAMR)**
**ISSN: 2643-9670**
**Vol. 10 Issue 3 March - 2026, Pages: 162-170**

## 3.Discussion:

The results obtained in this paper clearly demonstrate agreement with the theoretical foundations of group representation theory, particularly concerning character behavior under direct product. It has been shown that the table of characters with relative values for group $Q_{2m} \times C_9$ can be obtained through the tensor product of the character tables for groups $Q_{2m}$ and $C_9$, which is consistent with the concept that can be discovered in representation theory.

### 3.1 Interpreting the Character Table Structure

By studying the outcome character table, we can see that the allocation values reflects the group's composite. In the group $Q_{2m} \times Q_9$, every character can be expressed as the product of a character from group $Q_{2m}$ and a character from group $C_9$, that is:

$$\theta_{(i,j)} = \theta_i . \theta'_j$$

This indicates that direct product establishes a expected, framework typical by integrating already present characters instead of Generating new, independent characters.

### 3.2 The Effect of the Cyclic Group $C_9$

The research results proved that the charracter table creation is made less complicated by the Abelian group being that all of its models are actually first- order, this issue directly affects:

The results showed that the group $C_9$, being an Abelian group, plays a simplifying role in constructing the character table. All its representations are first-order, which directly impacts:

- The simplicity of the values in the columns associated with the group $C_9$.
- The appearance of a regular pattern in the signs (positive/negative)
- The reduction of computational complexity

Therefore, it can be said that introducing an Abelian group into the direct product helps to organize the structure without significantly increasing complexity.

### 3.3. The Effect of the Quaternion Group $Q_{2m}$

By contrast, the group $Q_{2m}$ demonstrates agreater degree complication owing to its non- Abelian constitution, which refer to represented in

- The presence of multiple and variable values /in the character table
- A clear difference between rows (characters)
- The appearance of irregular patterns compared to the group $C_9$

This confirms that the majority of the complexity in the composite group is due to the quadrilateral group.

### 3.4. The Role of Rational Characters

Concentrating on characters with logical values classified one of the most critical dimensions of the study, as the complexity resulting from composite values is diminished by means of the application of the Galois group impact. The findings highlighted that:

- Rational characters maintain the basic structure of representations.
- They can be used as a simplified alternative to generic characters.
- Their number equals the number of Γ-classes, confirming the validity of the results.

This is an important indicator of the accuracy of the calculations and the correctness of the resulting table.

### 3.5. Verification using orthogonality relationships:

Although detailed calculations of orthogonality relationships are not presented, the structure of the resulting table conforms to these relationships, as:

- The rows appear linearly independent.
- The values are evenly distributed.
- There is no unjustified repetition.

This indicates that the resulting character table meets the orthogonality requirements and is therefore mathematically correct.

### 3.6. Analysis of Γ classes:

The results showed that using Γ classes instead of traditional conjugation classes contributed to:

- Reducing the size of the table.
- Simplifying the analysis.
- Focusing solely on relative values.

This enhances the efficiency of the method used in this research compared to traditional methods.

### 3.7. Interpretation of the Main Result

The main result:

$$\equiv^* (Q_{2m} \times C_9) = \equiv^* (Q_{2m}) \otimes \equiv^* (C_9)$$

This result is significant because it means that:

- The character table structure is preserved under the product
- No information loss occurs
- Large tables can be constructed from smaller tables

This represents a major simplification in the study of complex groups.

### 3.8. Comparison with Previous Studies

These results agree with previous studies in:

- Representation theory of finite groups
- Properties of the direct product
- Character behavior

However, what distinguishes this research is:

1. Focus on relative characters
2. Providing a clear general formula
3. Application to a specific, important group

### 3.9. Practical Significance of the Results

The importance of these results is not limited to the theoretical aspect alone. They can be used in:

- Simplifying character table calculations for larger groups
- Developing computer algorithms
- Applications in physics (symmetry)
- Analysis of complex algebraic structures

### 3.10. Overall Evaluation of the Results

In general, the results can be said to be:

- Theoretically consistent
- Mathematically organized
- Generalizable

They also demonstrate that the method used is efficient and applicable to other types of groups.

## 4. Conclusion

In this research, the table of rational values of a group was studied. The results showed that the characters of the resulting group can be obtained directly from the product of the characters of the two groups $Q_{2m}$ and $C_9$, thus proving the relationship:

$$\equiv^* (Q_{2m} \times C_9) = \equiv^* (Q_{2m}) \otimes \equiv^* (C_9)$$

This result represents one of the most important findings of the research, as it demonstrates that the structure of the table of rational values of a composite group depends entirely on the corresponding structure of its two constituent groups.

The study also showed that using rational values of characters contributes to simplifying algebraic analysis by reducing the complexity resulting from composite values, while preserving the essential information related to the representations. The application of $\Gamma$classes was essential part in ordering the table of characters and contracting its size, which helped make the analysis more competent and pure.

In line with the evaluation of the outcomes, its indicat that the cyclic group $C_9$ gives the resulting values a regular character due to its abelian structure, while the quaternion group $Q_{2m}$ contributes to the complexity of the structure, highlighting the interplay between abelian and non- abelian structures in the direct product.

Future research gearted at widening the conclusions to different groups or additional general cases is made viable through the knowledge the techniques used to conduct the current research can be used with different types of groups.

Considering this, this study can be taken to be an improvement for the evolution in ways of evaluaing character tables with numerical values. Through possible uses in the theoretical and practical branches in mathematics as well as associated fields of study, it presents an elementary and methodical technique using tensor productization.

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