International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 10 Issue 4 April - 2026, Pages: 141-144

raph Based Generalized Approximation Spaces

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Abstract: *This study introduces a novel framework for generalized approximation spaces induced by graph structures. By defining dual neighborhood functions for each vertex, we construct new approximation operators and corresponding graph-induced spaces. The paper explores lower and upper approximations, boundary regions, and accuracy measures, offering a rigorous mathematical foundation. This approach extends classical rough set theory to graph-theoretic contexts, providing new insights into network analysis and discrete structures.*

Subject Classification (2010):05C30

Keywords— graph , $*T$ −space, $1T$ −space, $2T$ −space .

Introduction and Preliminaries

Graph theory and topology are two fundamental branches of modern mathematics that have attracted considerable attention due to their wide range of theoretical developments and practical applications. In recent years, the interaction between graph theory and topological structures has produced several important frameworks that allow researchers to study discrete systems using topological concepts. These approaches provide powerful tools for modeling relationships, analyzing uncertainty, and studying complex networks.[1,2,3,4,5,6,7] One of the most important mathematical tools for dealing with uncertainty is rough set theory, which was originally introduced by Pawlak.[8] Rough set theory provides a mathematical framework for approximating vague or incomplete information by means of lower and upper approximations. These approximations describe the boundary between elements that certainly belong to a set and those that possibly belong to it. Due to its flexibility and effectiveness, rough set theory has been widely applied in several areas such as data analysis, machine learning, information systems, and decision making. [9] Recently, several researchers have attempted to combine rough set concepts with graph structures in order to study uncertainty within networks. In such models, vertices represent objects while edges describe relationships between them. The concept of vertex neighborhood plays a fundamental role in describing the local structure of a graph and provides a natural way to construct approximation operators. [10,11] The use of neighborhood systems in graphs allows the construction of different approximation spaces, which can then be used to define various types of lower and upper approximations. These approximations help describe the structure of subsets of vertices and provide a way to measure the degree of certainty of information contained in graph-based systems. Motivated by these ideas, this work introduces new approximation operators induced by vertex neighborhood systems in graphs. The proposed framework constructs generalized approximation spaces derived from graph structures and investigates the associated lower and upper approximations. Furthermore, several fundamental properties of these approximations are established, and the concept of accuracy of approximation is studied within the proposed graph-based model. The results presented in this paper contribute to strengthening the connection between graph theory, topology, and rough set theory, and provide new tools for analyzing uncertainty in graph-based systems. These developments may also open new directions for future research in topological graph structures and rough graph models. Let $\mathcal{H} = (\mathcal{A}(\mathcal{H}), \mathcal{B}(\mathcal{H}))$ be a graph, where $\mathcal{A}(\mathcal{H})$ denotes the set of vertices and $\mathcal{B}(\mathcal{H})$ denotes the set of edges. An edge is an unordered pair of vertices representing a connection between them.[1] The neighborhood of $\mathcal{O} \in \mathcal{B}(\mathcal{H})$ is defined by $\boldsymbol{Nbh}(\mathcal{O}) = \{\mathfrak{m} \in \mathcal{B}(\mathcal{H}); (\mathcal{O}, \mathfrak{m}) \in \mathcal{B}(\mathcal{H})\}$.[2]. Rough set theory can be thought of as a new mathematical tool for analyzing flawed data. The hypothesis has been used in a variety of fields, including engineering, medicine, and others. The lower and upper approximations are a pair of precise sets that can be used with any rough set. The approximation of lower and upper bounds of a set is the formal classification of knowledge in the interest domain, and the approximation of spaces is the formal classification of knowledge in the interest domain. The partition characterizes a topological space, called approximation space $\mathcal{G} = (X, R)$, where X is a set called the universe and $R \subseteq X \times X$ is an indiscernibility equivalence relation[11,12]. The equivalence classes of R are also known granules, elementary sets or blocks. we will use $R_x \subseteq X$ to denoted the equivalence class containing $x \in X$.

Definition 1.1. [8] Let $\mathcal{G} = (X, R)$ be an approximation space and $A \subseteq X$, then the lower approximation (resp . upper approximation) of A is denoted by $L(A)$ (resp . $U(A)$) and is defined by :

International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
Vol. 10 Issue 4 April - 2026, Pages: 141-144

$$L(A) = \{x \in X; R_x \subseteq X\}$$

(resp . $U(A) = \{x \in X; R_x \cap A \neq \emptyset\}$)

Based on the lower and upper approximations of a set $A \subseteq X$, the universe X can be divided into three disjoint regions , the positive region (briefly $POS_R(A)$) , negative region (briefly $NEG_R(A)$) and boundary region (briefly $Bd_R(A)$) . Boundary , positive and negative regions are defined by :
$Bd_R(A) = U(A) - L(A)$,
$POS_R(A) = L(A)$ and
$NEG_R(A) = X - L(A)$.
The difference between the upper and lower approximations defines the set's boundary region. The definitions of the upper and lower approximations are shown in Figuree 1.1 [9].

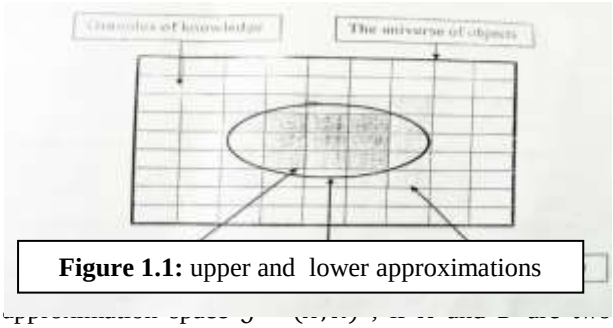

Figure 1.1: upper and lower approximations

An approximation space $S = (X, \tau)$, A and B are two subsets of X , then directly from the definitions of upper and lower and approximations , we can get the following properties of lower and upper approximations [8] :

(1) $L(A) \subseteq A \subseteq U(A)$,

(2) $L(\emptyset) = \emptyset, L(X) = X$ and $U(\emptyset) = \emptyset, U(X) = X$,

(3) U(A ∪ B) = U(A) ∪ U(B),

(4) L(A ∩ B) = L(A) ∩ L(B) ,

(5) If A ⊆ B , then L (A) ⊆ L (B) ,

(6) If A ⊆ B , then U(A) ⊆ U(B) ,

(7) L(A ∪ B) ⊇ L(A) ∪ L(B),

(8) U(A ∩ B) ⊆ U(A) ∩ U(B),

(9) $L(A^c) = [\mathrm{U}(\mathrm{A})]^c$,

(10) $U(A^c) = [\mathrm{L}(\mathrm{A})]^c$,

(11) $L(L(A) = U(L(A)) = L(A)$ and

(12) $U\big(U(A)\big) = L\big(U(A)\big) = U(A)$.

Main Results

Definition 2.1. Let $\mathrm{Ⱪ} = \big(\mathcal{A}(\mathrm{Ⱪ}), \mathcal{B}(\mathrm{Ⱪ})\big)$ be a graph and a vertex $\mathcal{O} \in \mathcal{A}(\mathrm{Ⱪ})$.

(i) The neighborhood set of $\mathcal{O}$ is denoted by $\mathcal{NHS}(\mathcal{O})$ and defined by: $\boldsymbol{\mathcal{NHS}(\mathcal{O})} = \{\boldsymbol{\mathcal{b}} \in \boldsymbol{\mathcal{A}}(\mathrm{Ⱪ}) |\ \boldsymbol{\mathcal{Ob}} \in \boldsymbol{\mathcal{B}}(\mathrm{Ⱪ})\}$.

(ii) The non-neighborhood set of $\mathcal{O}$ is denoted by $\mathcal{NHS}(\mathcal{O})^{\mathcal{C}}$ and defined by: $\big(\boldsymbol{\mathcal{NHS}(\mathcal{O})}\big)^{\mathcal{C}} = \{\boldsymbol{\mathcal{b}} \in \boldsymbol{\mathcal{A}}(\mathrm{Ⱪ}) |\ \boldsymbol{\mathcal{Ob}} \notin \boldsymbol{\mathcal{B}}(\mathrm{Ⱪ})\}$.

Definition 2.2. Let $\mathrm{Ⱪ} = \big(\mathcal{A}(\mathrm{Ⱪ}), \mathcal{B}(\mathrm{Ⱪ})\big)$ be a graph and let $\mathcal{P}\big(\mathcal{A}(\mathrm{Ⱪ})\big)$ denote the power set of $\mathcal{A}(\mathrm{Ⱪ})$. Assume that with each vertex $\mathcal{O} \in \mathcal{A}(\mathrm{Ⱪ})$ there are associated two subsets $\boldsymbol{\mathcal{NHS}(\mathcal{O})} \subseteq \mathcal{A}(\mathrm{Ⱪ})$ and $\boldsymbol{\mathcal{NHS}(\mathcal{O})^{\mathcal{C}}} \subseteq \mathcal{A}(\mathrm{Ⱪ})$. Define a mapping:
$\psi^{*T}: \mathcal{A}(\mathrm{Ⱪ}) \longrightarrow \mathcal{P}\left(\mathcal{P}\big(\mathcal{A}(\mathrm{Ⱪ})\big)\right)$ by: $\psi^{*T}(\mathcal{O}) = \left\{\mathcal{NHS}(\mathcal{O}), \big(\boldsymbol{\mathcal{NHS}(\mathcal{O})}\big)^{\mathcal{C}}\right\}$, $\forall \mathcal{O} \in \mathcal{A}(\mathrm{Ⱪ})$.
Moreover, consider the following two mappings:
$\psi^{1T}: \mathcal{A}(\mathrm{Ⱪ}) \longrightarrow \mathcal{P}\big(\mathcal{A}(\mathrm{Ⱪ})\big)$, $\psi^{1T}(\mathcal{O}) = \mathcal{NHS}(\mathcal{O})$,
$\psi^{2T}: \mathcal{A}(\mathrm{Ⱪ}) \longrightarrow \mathcal{P}\big(\mathcal{A}(\mathrm{Ⱪ})\big)$,$\psi^{2T}(\mathcal{O}) = \big(\mathcal{NHS}(\mathcal{O})\big)^{\mathcal{C}}$
Accordingly, the following graph spaces are induced:
1) The pair $(\mathrm{Ⱪ}, \psi^{*T})$ is called $*T$ −space,
2) The pair $(\mathrm{Ⱪ}, \psi^{1T})$ is called $1T$ −space,
3) The pair $(\mathrm{Ⱪ}, \psi^{2T})$ is called $2T$ −space.

Definition 2.3. Let $\mathrm{Ⱪ} = \big(\mathcal{A}(\mathrm{Ⱪ}), \mathcal{B}(\mathrm{Ⱪ})\big)$ be a graph, and for each $\mathcal{O} \in \mathcal{A}(\mathrm{Ⱪ})$, and let $\psi^{*T} \subseteq \mathcal{P}\left(\mathcal{P}\big(\mathcal{A}(\mathrm{Ⱪ})\big)\right)$. Define a binary relation $\mathcal{R}_{*T} \subseteq \mathcal{A}(\mathrm{Ⱪ}) \times \mathcal{A}(\mathrm{Ⱪ})$ by: $\mathcal{O}_1 \mathcal{R}_{*T} \mathcal{O}_2 \Leftrightarrow \mathcal{O}_1 \in$ Ⱪ where $\mathrm{Ⱪ} \in \psi^{*T}(\mathcal{O}_2)$. Then the ordered pair $\mathcal{S}_{*T} = (\mathcal{A}(\mathrm{Ⱪ}), \mathcal{R}_{*T})$ is called the generalized approximation space induced by the $*T$ −space.

Definition 2.4. Let $\mathrm{Ⱪ} = \big(\mathcal{A}(\mathrm{Ⱪ}), \mathcal{B}(\mathrm{Ⱪ})\big)$ be a graph, and for each $\mathcal{O} \in \mathcal{A}(\mathrm{Ⱪ})$, and let $\psi^{1T} \subseteq \mathcal{P}\big(\mathcal{A}(\mathrm{Ⱪ})\big)$. Define a binary relation $\mathcal{R}_{1T} \subseteq \mathcal{A}(\mathrm{Ⱪ}) \times \mathcal{A}(\mathrm{Ⱪ})$ by: $\mathcal{O}_1 \mathcal{R}_{1T} \mathcal{O}_2 \Leftrightarrow \mathcal{O}_1 \in \psi^{1T}(\mathcal{O}_2)$. Then the ordered pair $\mathcal{S}_{1T} = (\mathcal{A}(\mathrm{Ⱪ}), \mathcal{R}_{1T})$ is called the generalized approximation space induced by the $1T$ −space.

Definition 2.4. Let $\mathrm{Ⱪ} = \big(\mathcal{A}(\mathrm{Ⱪ}), \mathcal{B}(\mathrm{Ⱪ})\big)$ be a graph, and for each $\mathcal{O} \in \mathcal{A}(\mathrm{Ⱪ})$, and let $\psi^{2T} \subseteq \mathcal{P}\big(\mathcal{A}(\mathrm{Ⱪ})\big)$. Define a binary relation $\mathcal{R}_{2T} \subseteq \mathcal{A}(\mathrm{Ⱪ}) \times \mathcal{A}(\mathrm{Ⱪ})$ by: $\mathcal{O}_1 \mathcal{R}_{2T} \mathcal{O}_2 \Leftrightarrow \mathcal{O}_1 \in \psi^{2T}(\mathcal{O}_2)$. Then the ordered pair $\mathcal{S}_{2T} = (\mathcal{A}(\mathrm{Ⱪ}), \mathcal{R}_{2T})$ is called the generalized approximation space induced by the $2T$ −space.

Definition 2.5. Let $\mathcal{S}_{jT}$, where $j \in \{1,2,*\}$ be a generalized approximation spaces. For any $\mathcal{J} \subseteq \mathrm{Ⱪ}$, then:

1) The first jT-lower and jT-upper approximations of $\mathcal{J}$ where $j \in \{1,2,*\}$ are defined respectively by:
$L^1_{jT}\big(\mathcal{A}(\mathcal{J})\big) = \{\mathcal{O} \in \mathcal{A}(\mathcal{J}) : \psi^{jT}(\mathcal{O}) \subseteq \mathcal{A}(\mathcal{J})\}$,
$U^1_{jT}\big(\mathcal{A}(\mathcal{J})\big) = \mathcal{A}(\mathcal{J}) \cup \{\mathcal{O} \in \mathcal{A}(\mathrm{Ⱪ}) \setminus \mathcal{A}(\mathcal{J})$
$: \psi^{jT}(\mathcal{O}) \cap \mathcal{A}(\mathcal{J}) \neq \emptyset\}$,

International Journal of Academic and Applied Research (IJAAR)
ISSN: 2643-9603
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2) The first jT- boundary, jT- positive and jT- negative regions of $\mathcal{J}$ where $j \in \{1,2,*\}$ are defined respectively by:

$$Bnd^1_{jT}(\mathcal{A}(\mathcal{J})) = U^1_{jT}(\mathcal{A}(\mathcal{J})) \setminus L^1_{jT}(\mathcal{A}(\mathcal{J})),$$
$$POS^1_{jT}(\mathcal{A}(\mathcal{J})) = L^1_{jT}(\mathcal{A}(\mathcal{J})),$$
$$NEg^1_{jT}(\mathcal{A}(\mathcal{J})) = \mathcal{A}(\mathcal{H}) - U^1_{jT}(\mathcal{A}(\mathcal{J})).$$

Definition 2.6. Let $\mathcal{S}_{jT}$, where $j \in \{1,2,*\}$ be a generalized approximation spaces. For any $\mathcal{J} \subseteq \mathcal{H}$, then the first accuracy of the approximation of $\mathcal{J}$ is defined by:

$$\eta^1_{jT}(\mathcal{A}(\mathcal{J})) = 1 - \frac{|Bnd^1_{jT}(\mathcal{A}(\mathcal{J}))|}{|\mathcal{A}(\mathcal{H})|}$$

It is obvious that $0 \le \eta^1_{jT}(\mathcal{A}(\mathcal{J})) \le 1$, Moreover, if $\eta^1_{jT}(\mathcal{A}(\mathcal{J})) = 1$ then $\mathcal{J}$ is called $\mathcal{J}$-definable ($\mathcal{J}$-exact) graph otherwise, it is called $\mathcal{J}$-rough.

Example 2.7. Let $\mathcal{H} = (\mathcal{A}(\mathcal{H}), E(\mathcal{H}))$ such that $\mathcal{A}(\mathcal{H}) = \{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4, \mathcal{O}_5\}$ and

$$\mathcal{B}(\mathcal{H}) = \{(\mathcal{O}_1, \mathcal{O}_2), (\mathcal{O}_1, \mathcal{O}_4), (\mathcal{O}_2, \mathcal{O}_2), (\mathcal{O}_2, \mathcal{O}_3),$$
$$(\mathcal{O}_2, \mathcal{O}_4), (\mathcal{O}_2, \mathcal{O}_5), (\mathcal{O}_3, \mathcal{O}_4), (\mathcal{O}_4, \mathcal{O}_5), (\mathcal{O}_5, \mathcal{O}_5)\}.$$

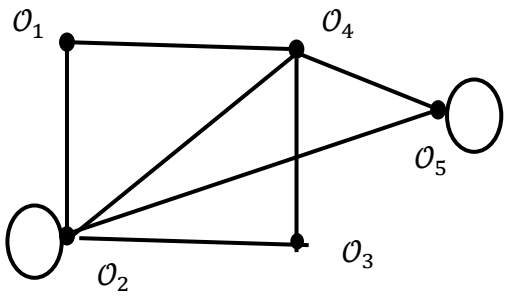

Figure 2.1: graph $\mathcal{H}$ given in Example (2.7).

We get:

$$\psi^{1T}(\mathcal{O}_1) = \{\mathcal{O}_2, \mathcal{O}_4\},$$
$$\psi^{1T}(\mathcal{O}_2) = \mathcal{A}(\mathcal{H}),$$
$$\psi^{1T}(\mathcal{O}_3) = \{\mathcal{O}_2, \mathcal{O}_4\},$$
$$\psi^{1T}(\mathcal{O}_4) = \{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_5\},$$
$$\psi^{1T}(\mathcal{O}_5) = \{\mathcal{O}_2, \mathcal{O}_4, \mathcal{O}_5\}.$$

Also we have:

$$\psi^{2T}(\mathcal{O}_1) = \{\mathcal{O}_3, \mathcal{O}_5\},$$
$$\psi^{2T}(\mathcal{O}_2) = \emptyset,$$
$$\psi^{2T}(\mathcal{O}_3) = \{\mathcal{O}_1, \mathcal{O}_5\},$$
$$\psi^{2T}(\mathcal{O}_4) = \emptyset,$$
$$\psi^{2T}(\mathcal{O}_5) = \{\mathcal{O}_1, \mathcal{O}_3\}.$$

Then we obtain:

$$\psi^{*T}(\mathcal{O}_1) = \{\{\mathcal{O}_2, \mathcal{O}_4\}, \{\mathcal{O}_3, \mathcal{O}_5\}\},$$
$$\psi^{*T}(\mathcal{O}_2) = \{\mathcal{A}(\mathcal{H}), \emptyset\},$$
$$\psi^{*T}(\mathcal{O}_3) = \{\{\mathcal{O}_2, \mathcal{O}_4\}, \{\mathcal{O}_1, \mathcal{O}_5\}\},$$
$$\psi^{*T}(\mathcal{O}_4) = \{\{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_5\}, \emptyset\},$$
$$\psi^{*T}(\mathcal{O}_5) = \{\{\mathcal{O}_2, \mathcal{O}_4, \mathcal{O}_5\}, \{\mathcal{O}_1, \mathcal{O}_3\}\}.$$

Accordingly, can be obtain the following table

Table 2.1 $\eta^1_{jT}(\mathcal{A}(\mathcal{J}))$, where $j \in \{1,2,*\}$ for all $\mathcal{J} \subseteq \mathcal{H}$.

$\mathcal{A}(\mathcal{J})$	$\eta^1_{1T}(\mathcal{A}(\mathcal{J}))$	$\eta^1_{2T}(\mathcal{A}(\mathcal{J}))$	$\eta^1_{*T}(\mathcal{A}(\mathcal{J}))$
$\{\mathcal{O}_1\}$	2/5	2/5	4/5
$\{\mathcal{O}_2\}$	0	1	1
$\{\mathcal{O}_3\}$	2/5	2/5	4/5
$\{\mathcal{O}_4\}$	0	1	1
$\{\mathcal{O}_5\}$	2/5	2/5	4/5
$\{\mathcal{O}_1, \mathcal{O}_2\}$	0	2/5	2/5
$\{\mathcal{O}_1, \mathcal{O}_3\}$	1/5	2/5	3/5
$\{\mathcal{O}_1, \mathcal{O}_4\}$	0	2/5	2/5
$\{\mathcal{O}_1, \mathcal{O}_5\}$	1/5	2/5	3/5
$\{\mathcal{O}_2, \mathcal{O}_3\}$	0	2/5	2/5
$\{\mathcal{O}_2, \mathcal{O}_4\}$	0	1	1
$\{\mathcal{O}_2, \mathcal{O}_5\}$	0	2/5	2/5
$\{\mathcal{O}_3, \mathcal{O}_4\}$	0	2/5	2/5
$\{\mathcal{O}_3, \mathcal{O}_5\}$	1/5	2/5	3/5
$\{\mathcal{O}_4, \mathcal{O}_5\}$	0	2/5	2/5
$\{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3\}$	0	2/5	2/5
$\{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_4\}$	1/5	2/5	3/5
$\{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_5\}$	0	2/5	2/5
$\{\mathcal{O}_1, \mathcal{O}_3, \mathcal{O}_4\}$	0	2/5	2/5
$\{\mathcal{O}_1, \mathcal{O}_3, \mathcal{O}_5\}$	0	1	1
$\{\mathcal{O}_1, \mathcal{O}_4, \mathcal{O}_5\}$	0	2/5	2/5
$\{\mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4\}$	1/5	2/5	3/5
$\{\mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_5\}$	0	2/5	2/5
$\{\mathcal{O}_2, \mathcal{O}_4, \mathcal{O}_5\}$	1/5	2/5	3/5
$\{\mathcal{O}_3, \mathcal{O}_4, \mathcal{O}_5\}$	0	2/5	2/5
$\{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4\}$	1/5	2/5	4/5
$\{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3\mathcal{O}_5\}$	0	1	1
$\{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_4\mathcal{O}_5\}$	2/5	2/5	4/5
$\{\mathcal{O}_1, \mathcal{O}_3, \mathcal{O}_4\mathcal{O}_5\}$	0	1	1
$\{\mathcal{O}_2, \mathcal{O}, \mathcal{O}_4\mathcal{O}_5\}$	2/5	2/5	4/5

$\mathcal{A}(\mathcal{H})$	1	1	1
$\emptyset$	1	1	1

Theorem 2.8. Let $\mathcal{S}_{jT}$, where $j \in \{1,2,*\}$ be a generalized approximation spaces. For any $\mathcal{J} \subseteq \mathcal{H}$,. Then:

(a) $L^1_{*T}(\mathcal{A}(\mathcal{J})) = L^1_{1T}(\mathcal{A}(\mathcal{J})) \cup L^1_{2T}(\mathcal{A}(\mathcal{J}))$
(b) $U^1_{*T}(\mathcal{A}(\mathcal{J})) = U^1_{1T}(\mathcal{A}(\mathcal{J})) \cap U^1_{2T}(\mathcal{A}(\mathcal{J}))$
(c) $\boldsymbol{Bnd^1_{*T}(\mathcal{A}(\mathcal{J})) = Bnd^1_{1T}(\mathcal{A}(\mathcal{J})) \cap Bnd^1_{2T}(\mathcal{A}(\mathcal{J}))}$
(d) $\boldsymbol{\eta^1_{*T}(\mathcal{A}(\mathcal{J})) \geq max\{\eta^1_{2T}(\mathcal{A}(\mathcal{J})), \eta^1_{1T}(\mathcal{A}(\mathcal{J}))\}}$.

Theorem 2.9. Let $\mathcal{S}_{jT}$, where $j \in \{1,2,*\}$ be a generalized approximation spaces. For any $\mathcal{J} \subseteq \mathcal{H}$,. Then:

(L_1) $L^1_j(\mathcal{A}(\mathcal{J})) \subseteq \mathcal{A}(\mathcal{J})$,
(L_2) $L^1_j(\mathcal{A}(\mathcal{H})) = \mathcal{A}(\mathcal{H})$,

(L_3) $L^1_j(\emptyset) = \emptyset$,
(L_4) if $\mathcal{A}(\mathcal{J}) \subseteq \mathcal{A}(\mathcal{T})$,then$L^1_j(\mathcal{A}(\mathcal{J})) \subseteq L^1_j(\mathcal{A}(\mathcal{T}))$
(L_5) $L^1_j(\mathcal{A}(\mathcal{J}) \cap \mathcal{A}(\mathcal{T})) \subseteq L^1_j(\mathcal{A}(\mathcal{J})) \cap L^1_j(\mathcal{A}(\mathcal{T}))$
$(L_6)L^1_j(\mathcal{A}(\mathcal{J}) \cup \mathcal{A}(\mathcal{T})) \supseteq L^1_j(\mathcal{A}(\mathcal{J})) \cup L^1_j(\mathcal{A}(\mathcal{T}))$ (L_7)
$L^1_j(\mathcal{A}(\mathcal{J})) = \mathcal{A}(\mathcal{H}) - [U^1_j(\mathcal{A}(\mathcal{H}) - \mathcal{A}(\mathcal{J}))]$ (U_1) $\mathcal{A}(\mathcal{J}) \subseteq U^1_j(\mathcal{A}(\mathcal{J}))$,
(U_2) $U^1_j(\mathcal{A}(\mathcal{H})) = \mathcal{A}(\mathcal{H})$,
(U_3) $U^1_j(\emptyset) = \emptyset$,
(U_4) if $\mathcal{A}(\mathcal{J}) \subseteq \mathcal{A}(\mathcal{T})$,then $U^1_j(\mathcal{A}(\mathcal{J})) \subseteq U^1_j(\mathcal{A}(\mathcal{T}))$
$(U_5)U^1_j(\mathcal{A}(\mathcal{J}) \cap \mathcal{A}(K)) \subseteq U^1_j(\mathcal{A}(\mathcal{J})) \cap U^1_j(\mathcal{A}(\mathcal{T}))$
$(U_6)U^1_j(\mathcal{A}(\mathcal{J}) \cup \mathcal{A}(\mathcal{T})) \supseteq U^1_j(\mathcal{A}(\mathcal{J})) \cup U^1_j(\mathcal{A}(\mathcal{T}))$ (U_7)
$U^1_j(\mathcal{A}(\mathcal{J})) = \mathcal{A}(\mathcal{H}) - [L^1_j(\mathcal{A}(\mathcal{H}) - \mathcal{A}(\mathcal{J}))]$
(LU) $L^1_j(\mathcal{A}(\mathcal{J})) \subseteq U^1_j(\mathcal{A}(\mathcal{J}))$.

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