

Trapezoidal Method Application

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Abstract: *Trapezoidal method* is a one of fundamental integration techniques in numerical analysis used to approximate the value of definite integrals. This paper demonstrated that the Trapezoidal rule is the mathematical derivation using linear interpolation, discusses the its capability to offer a good balance between simplicity, stability, and accuracy. As well as, method applications performance in the estimating the objects and tumours volume in the medical image processing.

Keywords: Trapezoidal rule, medical image, tumour volume

1. Introduction

The Trapezoidal al Method is an analysis numerical technique able to integrate the difficult function, estimates the value of definite integrals. The method approximates the area under a curve by dividing it into Trapezoidal s instead of rectangles. The inherent properties of the trapezoidal method render it particularly capable of addressing a wide range of image processing challenges, especially in the computation of area and volume measurements associated with image objects. This is achieved by calculating the integrated areas of objects across multiple sequential image planes and subsequently aggregating these areas to approximate a third dimension, thereby enabling the designed image processing model to estimate the volumetric properties of a given object [1].

2. Mathematical basic principle

Trapezoidal formula demonstrates the interval $[a, b]$ divided in subintervals within the area that is under curve $f(x)$. The figure (1) shows how the line segment, connects two adjacent points to form trapezoidal for each subinterval under curve, paralleled the x-axis.

For a single trapezoidal over $[a, b]$, the formula is [2]:

$$\int_a^b f(x) dx \approx (b - a) \cdot [f(a) + f(b)]/2$$

For the composite Trapezoidal al rule with n equally-spaced subintervals of width $h = (b - a)/n$:

$$\int_a^b f(x) dx \approx \frac{h}{2} \cdot [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)]$$

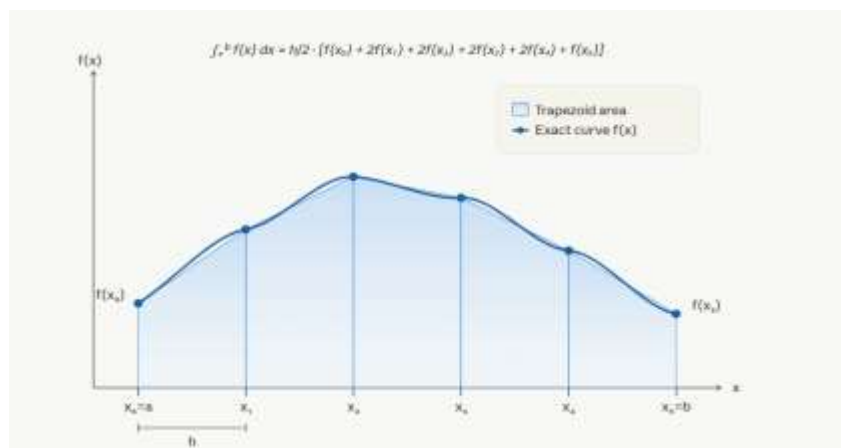


Figure (1): Trapezoidal areas under curve $f(x)$

3. Discussion

The Trapezoidal method — formally known in numerical analysis as the Trapezoidal rule — provides a robust mechanism for approximating the definite integral of a function by summing the areas of Trapezoids formed between consecutive sample points. In the context of medical image processing, this principle is directly applied to the problem of three-dimensional volume estimation from sequential two-dimensional image slices, such as those produced by Computed Tomography (CT) or Magnetic Resonance Imaging (MRI). Trapezoidal method application in [3], [4], [5], [6],[7],[8] serve the primary volume estimation in MRI or CT images and observe the volumetric changes after and before the surgery of tumor. It can compute the areas of the tumor that across the slices of MRI stack and multiply the summation of areas by the thickness value of tumor slices. Trapezoidal method proves as gold-standard reference compared with the other methods were validated in this process, to be well-suited to deal with 3D volume estimation for objects in the medical image processing.

4. Conclusions

This approach is applicable to sequential and cross-sectionally segmented image data, such as video sequences and medical imaging modalities including Computed Tomography (CT) and Magnetic Resonance Imaging (MRI). In the context of medical image analysis, tumor volume estimation can be performed by computing the cross-sectional area of a tumor across multiple slice levels within a medical image stack, by trapezoidal method. The successive areas obtained from each slice are then summed, and when combined with the corresponding slice thickness measurement, they yield a third-dimensional approximation that enables the model to accurately estimate the volumetric extent of the tumors.

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